EXPERIMENTAL INVESTIGATION OF PLASTIC DEFORMATIONS OF ARGILLACEOUS SOIL WITH TRIAXIAL COMPRESSION

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The article describes the change in the loading surface during the process of simple loading. A study is made of the direction of the vector of the increment in the plastic deformation at different points of the loading surface. The experimental data were obtained in tests on the controlled triaxial compression of hollow cylindrical samples of argillaceous soil. The applicability of the theory of increments in the form of an association law is established.

Calculations of the stress-deformation state of a soil, for foundations, media, and various structural materials, come down mainly to the use of the relationships of the deformation theory of plasticity. However, experimental investigations in recent years have shown that, with complex loading, the relationships of the deformation theory break down. In particular, a significant dependence of the trajectory of the deformations on the trajectory of the stresses has been demonstrated experimentally [1]. On the other hand, it is well known that, under natural conditions, the loading of a soil, as a rule, does not correspond to simple, or even close to simple, loading conditions. What has been said explains the present interest in the theory of the increment of the deformations. The experimental studies required to verify this theory and to give a concrete form to its equations are very limited [2, 3]; this refers in particular to experimental investigations with a triaxial state of stress.

The present investigations are based on the concept of surface loading and the associated law of plastic deformation, connecting the tensor of the increments of the plastic deformations de_{jk}^{p} with the tensor of the stresses S_{jk} by the relationship

$$de_{jk}^{p} = G\frac{\partial f}{\partial S_{jk}}df$$

where f is the loading function (it is the equation of the loading surface in the space of the increments).

Controlled triaxial compression of the soil was effected by the loading of hollow cylindrical samples using an axial force, as well as by hydrostatic pressures, different outside and inside of the sample. The vertical deformations were measured using dial-type indicators with a graduation of 0.01 mm, and the tangential and radial deformations from the changes in the inside and outside diameters of the sample, recorded by corresponding volumenometers, connected with the internal and external chambers of the experimental unit (the graduation of the tube of a volumenometer was 0.05 cm^3). The accuracy of the measurement of the vertical deformations was 0.01%, and of the tangential and radial, 0.03%. The height of the sample of soil was 80 mm, and the inside and outside diameters were 35 and 60 mm, respectively.

The kaolinite loam investigated has the following indices: mositure content at the yield point 30%; plasticity number 10, porosity coefficient 0.76; moisture by weight 12.3%; degree of moisture content 0.44. The initial state of the density and the moisture content were achieved by densification of previously moistened powder. The initial isotropy of the samples of soil was confirmed by special experiments. The latter consisted in the crushing of differently oriented samples following an identical program.

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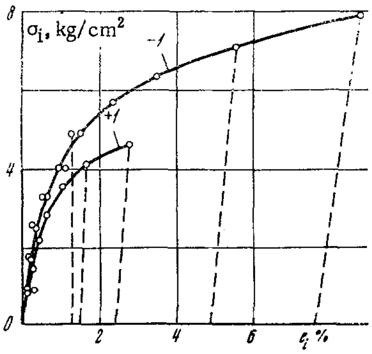

Fig. 1

The experiments were set up with a constant value of the mean pressure, $\sigma = 5.0$ kg/cm^2. Thus, the dependence of the plastic deformations on σ, which is very considerable for soils, was not taken into account in the present investigation. In addition, during the tests of a given cycle, the main axes of the stresses were fixed; therefore, with a given orientation of the axes, the change in the state of stress is characterized by the intensity of the stresses σ_i and by a parameter of the type of state of stress μ. The consideration of the loading surface is reduced to a study of its trace in a plane perpendicular to the axis $\sigma_1 = \sigma_2 = \sigma_3$ in the space of the main stresses.

The identical value $\sigma = 5.0$ kg/cm^2, given in the experiments, was achieved by hydrostatic compression of the samples. Further plastic deformation of the soil was carried out under conditions of radial loading with values of $\mu = -1, +1$. With $\mu = -1$, three loading surfaces were plotted, corresponding to different intensities of the deformations e_i, respectively, for 1.25, 5.6, and 8.5%. Two loading surfaces were investigated with $\mu = +1$, for values of e_i equal to 1.6 and 2.8%.

A majority of the points of each loading surface was plotted using one sample of soil in the experiments. The initial loading was stepwise, with stabilization of the deformations of the creep at each stage of an increase in σ_i. After the given program of the values of e_i had been completed, the sample was unloaded ($\sigma_i = 0$, $\sigma = 5.0$ kg/cm^2), and then again loaded stepwise, with different radial paths of the stress. As a criterion of the attainment of a loading surface, there was adopted an increment of 0.1% in the deformation of the sample in an axial direction. The final radial loading was carried out along the path of the original loading. This confirmed the absence of any significant shift of the surface as a result of the secondary loadings. Certain segments of the surfaces were plotted using different samples and, in this case, part of the points are duplicated. At some points of the surface, the loading was carried out in small steps in different directions, to study the orientation of the vector of the increment in the plastic deformation.

The experimental results are presented in the form of curves in Fig. 1 and Fig. 2. Figure 1 shows the character of the strengthening of the investigated soil during the process of primary loading along radial paths with $\mu = -1$ and $+1$ (the value of μ is noted on the curves). Rather great deformations of the soil are characteristic. The hardening of the investigated soil depends essentially on the type of the state of stress. The greatest degree of hardening is attained at $\mu = -1$. At the start of the loading, a small linear segment, common to both curves, is observed. During the process of further loading, the elastic properties of the soil vary only slightly, and the deformations are basically of an irreversible character (the loading lines are shown by dashes in Fig. 1).

The development of a loading surface during the process of plastic deformation, with $\mu = -1$ and $+1$, is shown in Fig. 2, a and b, respectively, in the form of a projection on the plane $\sigma = 5.0$ kg/cm^2. The surfaces α, β, γ (Fig. 2a) correspond to the plastic deformations $e_i = 1.25$, 5.6, and 8.5%, and the surfaces δ, ε (Fig. 2b) to the deformations $e_i = 1.6$ and 2.8%. The dot-dashed lines show the paths of the secondary radial stresses. The numeration of the points for each surface is individual, and corresponds to the sequence of the secondary stresses. The numbers with primes and without primes denote points obtained for different samples of soil. The dashed lines were drawn for segments of the surface not having experimental points, in accordance with the features of more fully investigated surfaces.

An examination of the curves shows that, as a result of plastic deformation, there is considerable deformation of the load surface, with its elongation in the direction of the load. The form of the surface differs very considerably from that of a Mises neighborhood. The degree of elongation of the surface depends on the direction of the loading, and is more sharply expressed for $\mu = -1$. The surfaces remain symmetrical with respect to the directions of the original radial loadings.

During the process of loading in one direction, there is an expansion of the surface in all directions. The segment of the surface adjacent to the direction of the primary loading is located considerably further from the origin of coordinates (point O) than the opposite segment of the surface. The surfaces are smooth and convex; no corners are formed on the surface.

The position of the projection of the vector of the increment of the plastic deformation on the plane $\sigma = 5.0$ kg/cm^2, at different points of the loading surfaces investigated, is shown in Fig. 2a and b. The thin

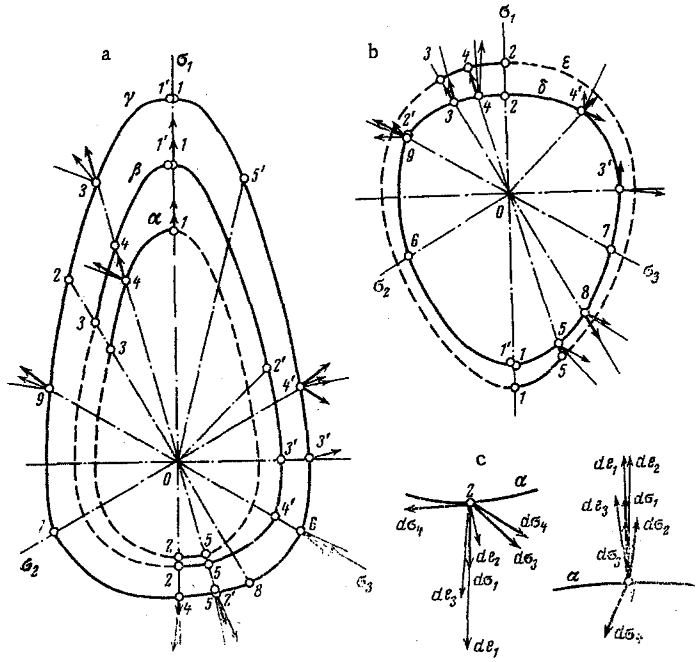

Fig. 2

solid line shows the direction of the normal to the loading surface, while the thick arrows show the vectors of the increment of the stresses, and the thin arrows the vectors of the increment of the plastic deformation.

An examination of the curves of Fig. 2a and b shows the following. Since the form of the loading surface differs considerably from a circle, the direction of the vector of the stress, as a rule, does not coincide with the direction of the normal to the surface. With intersection of the loading surface by the stress vector at an acute angle, and with subsequent loading, the vector of the increment of the plastic deformation de^p, in a majority of cases, coincides neither with the direction of the vector of the stress, nor with the direction of the vector of the loading $d\sigma$. In all cases, de^p deviates from $d\sigma$ toward the side of the normal to the loading surface, and is located closer to the latter than to $d\sigma$, sometimes coinciding exactly with the normal (points 4α* and 9γ in Fig. 2a; points 4δ, 8δ in Fig. 2b). Furthermore, the vectors of the increment of the plastic deformation, obtained with loading at points located on the axis of symmetry of the loading surface (points 1α, 1β, 4γ, in Fig. 2a) also coincide with the normal. The noncoincidence of de^p and $d\sigma$ follows also from the fact that, with loading, positive increments of the volumetric deformation are obtained, i.e., de^p not only does not coincide with the stress vector, but does not lie in the plane $\sigma =$ const, and is inclined to it. This circumstance, characteristic for soils, is evidence of the closed nature of the loading in the direction of the three-dimensional diagonal [3].

Figure 2c gives individually the vectors of the increment of the plastic deformations, obtained with loadings in several directions at the points 1 and 2 of the loading surface, α. The numbers denote the sequence of the loadings at each point. As before, de^p does not coincide with $d\sigma$. A change in the direction of $d\sigma$ changes the direction of de^p, but this change is less marked and the resultant vector de^p is located, as a rule, closer to the normal than to $d\sigma$. It must be noted that, with loadings directed along the tangent to the loading surface or within it ($d\sigma_4$ at points 1 and 2), an increment of the plastic deformations is not obtained. On the whole, an examination of the curves of Fig. 2 shows, without a doubt, that on the basis of the experimental data considered, the hypothesis of the normality of de^p to the loading surface is satisfactorily confirmed.

The experimental investigations of the plastic deformations of argillaceous soil, carried out under conditions of controlled triaxial compression, permit the following conclusions.

Deformation of the soil investigated is generally irreversible. Hardening of the soil during the process of plastic deformation is of a sharply marked directed character. During the process of radial loading,

*The letter with the number of the point denotes the index of the corresponding loading surface.

the loading surface expands symmetrically and is elongated in the direction of the radial loading, but without movement of the surface as a whole. The form of the surface depends on the direction of the primary loading. The loading surface is smooth and convex; there is no conicity at the loading point.

The direction of the vector of the increment in the plastic deformation is not determined by the vector of the stresses and does not coincide with it, as a result of the nonround form of the surface, with the exception of the case when the direction of the latter is close to the normal to the surfaces. In particular, with loading in the plane σ = const, there are increments in the volumetric deformation. The results of the investigations satisfactorily confirm the hypothesis of the singularity of the direction of the vector of the increment of the plastic deformation and of its normality to the loading surface. The loading surface of the soil investigated cannot be represented in the form of an isotropic function only of the intensity of the stresses, or of the three invariants of the stress, but must include the characteristic of the direction of the loading. The nonround form of the loading surface and the noncoincidence of the direction of the direction of the vector of the increment of the plastic deformations and the stress vector do not permit applying the simplest form of the associated law

$$de^{p}_{jk} = F(\sigma_i) S_{jk} d\sigma_i$$

However, on the basis of the experiments carried out, the concept of the loading surface and of the associated law of plastic deformation is completely acceptable.

The investigations demonstrate the utility of using the concepts of the theory of plasticity (in the present case, the theory of flow) in soil mechanics. In the theory of plasticity, along with the development of the principles of the general theory, a great deal of attention is being paid to the construction of the simplest mathematical models, within the framework of the phenomenological approach. In particular, detailed investigations in this direction have been made by M. Ya. Leonov et al. [4]. In these articles, a model for a linear anisotropic hardening medium is proposed, and a class of loadings is isolated, differing from proportional loadings, for which the application of the deformation theory is possible. The application of simpler mathematical theories, constructed on a phenomenological basis, and designed to describe limited classes of loadings, is doubtless feasible also in soil mechanics.

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